THE HALL EFFECT IN THE RECIPROCAL SYSTEM

Gopi Vijaya

2017

Introduction

The Hall Effect, first discovered in 1879 by Edwin Hall[1], is a prime example of a phenomenon that has remained at the center of physical theories and has yet proved puzzling to generations of scientists. The effect itself is straightforward to grasp. When a material has an electric potential difference between its two ends, and there is a magnetic field applied at right angles to the flow of electricity, an induced potential difference appears across the third perpendicular direction (Fig. 1.)

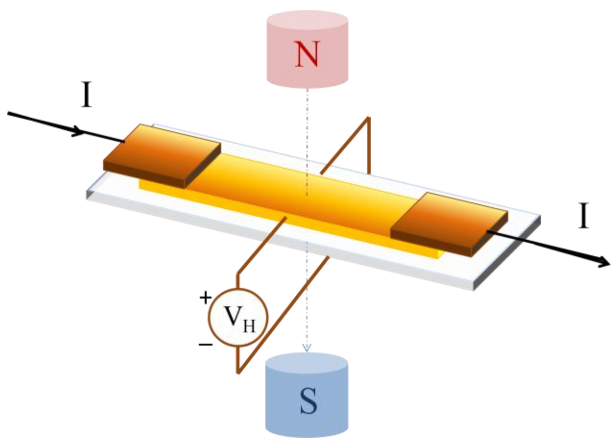

Fig. 1: Hall Voltage (V_H) developed across a current-carrying conductor in a magnetic field

This bears a distinct similarity to the perpendicularity of Fleming's Left Hand rule often used in engineering, with one crucial difference: The direction of the Hall voltage is *opposite* to what is expected by that rule. The founders of this effect, Hall and his mentor Rowland ran into this difficulty immediately[2]:

> Rowland's theoretical speculations also placed great emphasis on the *direction* of the Hall effect. Both men had expected the direction of the transverse e.m.f. to be the same as that of the ordinary force exerted on a current carrying conductor in a magnetic field. It was, however, found to be in the reverse direction for the first two metals examined, gold and silver.

In other words, the "Hall coefficient" of these metals was found to be negative. That was not all – in several metals (e.g. Fe, Cd, Zn) the Hall coefficient turned out to be *positive*. There were hence two puzzles at this point:

a) *Why is the sign of the Hall coefficient opposite to that expected?*
b) *Why do some materials have a positive coefficient?*

In his attempt to answer this question, Hall proposed positive ions as charge carriers[3]:

> Perhaps the one outstanding feature that distinguishes his theory from others was his constant insistence on the importance of the role of the positive ions in metallic conduction. The early electron theories were concerned almost entirely with the role of the free electrons. It was obvious enough that where there are free electrons there must be ions, but for some reason the role of these ions was not explored in the conventional theories, perhaps because of mathematical difficulties. In particular, Professor Hall saw and insisted on the importance of the positive ions in affording an explanation for the unexpected sign of the Hall effect.

This was a time when the role of electrons was unclear. It was only in 1897 that Thomson discovered electrons, and it was almost immediately assumed that only negative electrons have the role of charge carriers (experiments with metals[4] and the Drude Model[5]). Still, this assumption does not answer question (b) above, since for some metals the sign was positive. What are the charge carriers in this case?

Quantum Mechanics

The positive hall coefficient was still a puzzle when quantum mechanics was being developed in the 1920's. This was one of the reasons for Heisenberg to offer the problem as a dissertation topic to Peierls[6] in 1928:

> Heisenberg then suggested that Peierls look at the Hall effect, the buildup of a transverse voltage as an electric current passes through a metal in a magnetic field. Sommerfeld's semiclassical theory, based on free electrons, could not essentially improve on the classical result for the Hall voltage, which, although agreeing well with observation for the alkalis and certain other metals (copper, gold, silver, lead, palladium, and manganese), could not account for the variations of the Hall voltage with temperature or magnetic field or explain why for certain metals theory gave the wrong magnitude or sometimes even the wrong sign.[7]

Peierls had to find a way to bring in a change of sign. This was done by using an expression from quantum mechanics where the energy is given as a sinusoidal wave[8] with respect to the momentum *k*:

$$p = \text{const.}\ \sin\xi \quad \text{and}\ E = E_0 - \text{const.}\ \cos\xi, \ \text{ where } -\pi \leq \xi \leq \pi.$$

Essentially both momentum and energy have been used as oscillating quantities, out of phase with each other. This oscillation makes it possible to generate both a rise and a fall of something that is normally constant, such as energy. With this out-of-phase oscillation it was possible for the electron to *lose* energy as its velocity increased, a concept that implies *negative* energy increments. Peierls used this negative sign in his equations to flip the sign of the Hall coefficient, in essence hinging the explanation on the behavior of the energy-momentum relation.

The energy momentum curves are ultimately empirical, and the physical explanation for energy to decrease when momentum increases is generally not tackled. At this point it is difficult to calculate which materials must have a positive Hall coefficient, since the reason is hidden in an empirical graph. Calculations are also approximate since the three-body-problem is incalculable, and hence so is the crystal lattice dynamics.

To summarize, the answers of Quantum Mechanics to the second question (b) go this way:

> *b) Why do some materials have a positive coefficient?*
>
> A: Because their energy reduces with velocity.
>
> *c) Why does the energy behave like that?*
>
> A: It is just the net effect of the entire crystal, which cannot yet be calculated exactly.

This explanation survives to this day, more than a century after the identification of the Hall effect. This odd behavior of the energy is given the label "hole" and is treated as a positive *particle*, even though its definition is vague (absence of an electron, or a slope of a curve). This trend of treating a positive particle as the "absence of a negative particle" was continued with explanations such as the Dirac Sea, where the bubbles in the sea form the holes. A roundabout route is taken while directly identifying charge carriers as positive has become forbidden. This is probably because the only known positive charge carriers i.e. positrons are observed to annihilate with electrons to give light. Yet this very fact can lead on in a different direction, one that does not need to avoid the positive charge carrier.

Reciprocity of Electricity and Magnetism

Since the only known particle opposing the electron combines with it to create light, it is worth asking if there is any other behavior that acts opposite to electricity and does not cancel out an electron the way a positron does. It is easy to find the answer to that: *magnetism*. Magnetism is a true complement to electricity, since electric and magnetic fields do not annihilate each other, but they do have some features that are opposite. For example, while like charges *repel* each other, two conductors carrying current in the same direction *attract* each other.

According to the Reciprocal System, both electricity and magnetism have an independent existence[9], and it is not necessary for electricity to be primary and magnetism a dependent secondary effect. It may be a pointed out that traditional explanations are 'electron-biased,' i.e. they have a bias against positive current, there is also a bias against treating magnetism independent of electricity. Electricity is 1-dimensional, while magnetism is 2-dimensional. Additionally, the crystal lattice is made up of both electric and magnetic components, with the electric rotation being oppositely directed to the magnetic[10]:

> But after the two-dimensional rotation is in existence it is possible to give the entire combination of vibrational and rotational motions a rotation around the third axis, which is also inward from the scalar standpoint, but is opposed to the two-dimensional rotation vectorially. This reverse rotation is optional, as the basic rotation is distributed over all three dimensions, and nothing further is required for stability.

This means that when expressed in the Euclidean frame, the 2-D rotation is the solid rotation (4π) and the 1-D rotation is the regular one (2π), and they are *oppositely directed* i.e. if one is clockwise, the other is counter-clockwise and vice-versa.

Electricity is the flow of uncharged electrons in the crystal lattice, as the relation of rotational space (s) to the overall lattice (t) since space to time is motion. However, it still requires a potential difference for the flow to occur. A similar process must be possible for magnetism as well. A magnetic rotation, which is a 2D *temporal* displacement, only allows movement if it occurs through the *space* of the substance, which in this case is provided by the electric rotation. The equivalent of an emf for current is the presence of a magnetic field. The question then becomes: Can we create a movement of magnetic displacements simply by applying a magnetic field? The answer is no: because the lattice as a whole is temporal, and time/time is not motion, and even the presence of electric rotations provides the flow path only in random directions. It is only when the flow of electrons is streamlined and oriented, by applying an emf that the corresponding magnetic orientation also develops. Hence, the presence of a voltage and that of the magnetic field together are necessary (Fig. 2). The orthogonal relationship between the two fields continues as it does in other electro-magnetic phenomena.

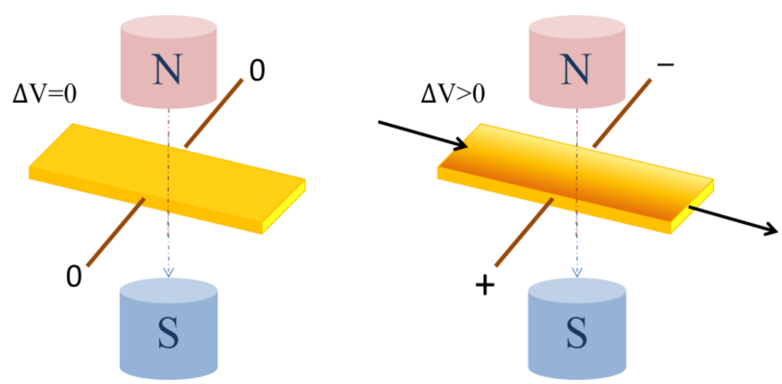

Fig. 2: A magnetic field alone is ineffective, an electric potential difference is necessary

Hence, just as the electric displacement 0-0-(1) in an electric potential generates the flow of electricity, the presence of a magnetic displacement 1-0-0 (or more accurately, ½-½-0) in a magnetic potential (field) allows the development of the Hall voltage. The reason for the opposite sign is just because it is temporal instead of spatial, and the Hall coefficient becomes positive. Since the movement of these 2D magnetic displacements or *magnons* for short (*not* magnetic monopoles since they are 2D) is more sensitive to the pathway formed for it in space, they only occur when the pathway conditions are met. Hence in the usual symmetric case, the force on the electrons causes a negative Hall coefficient, while in cases where the magnetic displacements exist and a pathway can be formed, they generate a positive Hall coefficient. The net effect is the resultant of these two opposite processes.

If this magnetic dependence of positive Hall coefficient is correct, then:

1. There must be a correlation between Hall coefficients and magnetic materials.
2. There must be a correlation between these displacements and "holes" of conventional theory.
3. Explain certain Quantum phenomena in Hall effect

Each of these topics will be picked up in turn.

1. Magnetic Materials

From the Reciprocal System, the effect of magnetic displacements is more predominant when, in the *a-b-c* notation, the "a-b" corresponding to the magnetic displacements are asymmetrical, e.g 2-1 or 4-3[11]. It is also helped by the presence of divalence, or even number of electric displacements, since magnons are 2D. When the number of electric displacements equals that of one full magnetic rotation i.e. 8 units, then ferromagnetism is more probable. So one would expect that the presence of elements with asymmetric a-b, and a large even 'c' (preferably c>7) is more conducive to the presence of positive Hall coefficient. This indeed, appears to be the case (Table 1.)[12,13]

Table 1: Hall Coefficients of Elements

(* - explained in text)

Element	RS notation (**a-b**-c)	Hall Coefficient ($10^{-5}cm^3/C$)
Be	**2-1**-2	+ 24.4
V	**3-2**-5	+ 7.6
Fe	**3-2**-8	+ 100.0
Co	**3-2**-9	+ 24.0
Ni*	**3-3**-(8)	- 60.0
Zn	**3-3**-(6)	+ 3.3
Zr	**3-3**-4	+ 13.0
Nb*	**3-3**-5	+ 9.0
Mo	**3-3**-6	+ 12.6
Ru	**3-3**-8	+ 22.0
Rh	**4-3**-(9)	+ 50.0
Cd	**4-3**-(8)	+ 6.0
Nd	**4-3**-6	+ 9.7
Yb	**4-3**-16	+ 38.0
W	**4-4**-(12)	+ 11.8

As can be seen, for almost all of the samples with positive coefficients, the necessary conditions are met. In the case of nickel (Ni), even though (8) electric displacement units (equal to 1 full magnetic unit) are available, they act in the opposite direction since the magnetic rotations are symmetrical 3-3. Hence this would have a large coefficient, due to magnetic effect, but in the opposite direction since it has its basis in the electric rotation. In niobium (Nb), the reason for the residual effect is unknown.

It must be mentioned that even though it was originally suspected that positive Hall coefficient is related to magnetism[2], they were not understood as such because the opposition with electric displacement, as well as the presence of independent magnetic rotations, were unknown. Thus, the magnetic approach was abandoned as soon as Ni was found to have a negative coefficient.

2. Hole theory

Many of the puzzling features of the so-called "holes" in conventional electronics will be solved by using the approach of magnons. Holes are connected with positive or "p-type semiconductors" while electrons are connected with "n-type semiconductors," where holes are treated like gaps in the presence of electrons. Yet, these gaps are still treated as positive particles with strange properties. For instance, it is hard to see how the effective mass of holes can be *negative*, and the usual approach that points at a curvature in a band diagram is very unsatisfactory for such a revolutionary idea. It is almost as if one pushes forward and the object moves backward. With the magnetic displacements, in their very structure they are opposite to electrical displacements, hence no "negative mass" has to be postulated. They are displacements in their own right allowing a magnetic flow, just as electric displacements allow electron flow.

Another puzzle has been the fact that the magnitude of hole "effective mass" has always been greater than that of the electron. Conventionally it has been very hard to explain why the *absence* of an electron behaves like the *presence* of a bigger electron. With the two-dimensional nature of the magnon, it is easy to see that the mass effect is considerably bigger than that of the one-dimensional electron. There are also 3 types of holes: heavy holes, light holes, and split off holes. There is no explanation for this phenomenon in the literature, but it is easy to see the connection with the solid rotation (Fig. 3.) In fact, the split-off band is also called the "spin-orbit" band, showing that there has been an attempt to connect it to 1D-2D revolutions, where spin is 2D and the traditional electron-orbit is planar 1D as shown in Fig. 3.

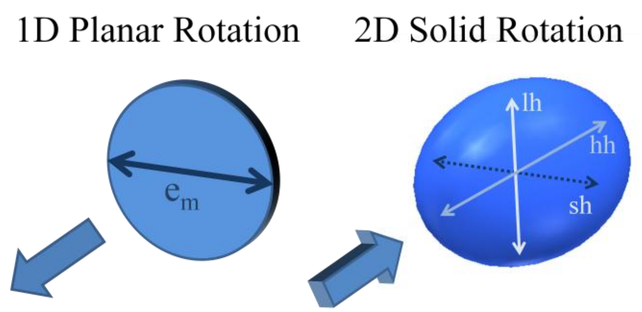

Fig. 3
Left: Electron rotation offering one surface as resistance to the flow
Right: Magnetic Rotation offering resistance in three directions i.e. heavy hole (hh) light hole (lh) and split off hole (sh) in the same plane but in opposite direction of electricity.

It also explains the graph used often in quantum mechanical band calculations (Fig. 4.) with energy plotted against momentum. The sign reversal and the three types of holes are indicated. Rather than using the curvature of the plot lines to determine a negative mass, it can even be predicted.

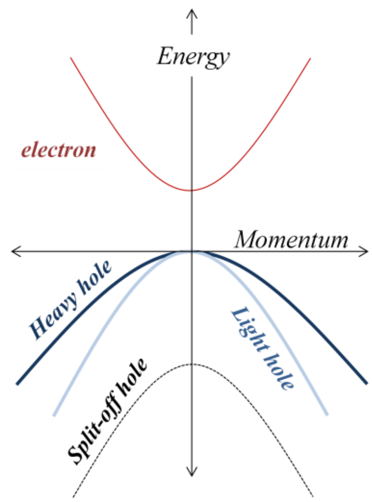

Fig. 4
Plot of energy vs momentum in a crystal lattice, showing the different energy levels. This is the prototype of all band-structure diagrams.

3. Quantum Phenomena

Along with the positive Hall coefficient, another surprising phenomenon has been the 'anomalous' Hall Effect, where the Hall coefficient has a contribution from the magnetization of the material e.g. in ferromagnets. By identifying magnetism as the basis of the positive coefficient, this effect is no longer anomalous, but expected. Since ferromagnetism is explained as a co-magnetic phenomenon[14], it is also expected that the contribution from this side have a second power relation to the regular magnetism. This has indeed been found in experiment[15], where the 'anomalous' coefficient is proportional to the square of the resistivity while the normal coefficient is proportional to the resistivity alone. Another observation – the 'Spin Hall effect' – can similarly be described as what happens to uncharged electrons when they are caught in the cross current of magneton flow. The spins (one dimensional rotations) get segregated in two opposite directions. This is exactly how spins are segregated in a magnetic field (giving spin up and spin down) except that this time it occurs in the interior of the material.

All the three effects – Hall effect, Anomalous Hall effect, and Spin Hall effect – have their "squeezed" versions when observed in materials less than one unit of space thick (called low-dimensional structures.) They are respectively the Quantum Hall Effect, Anomalous Quantum Hall Effect, and Quantum Spin Hall effect[16]. Since they are the result of dimensional reduction, they show properties that are discontinuous. The Quantum Hall effect has two types: integer and fractional versions, where the current rises in steps. The presence of only odd numbers in non-responsive sections in the fractional Quantum Hall effect is also further confirmation that only magnetic displacements (even) are involved in the Hall voltage. Since the electric displacements get highlighted here due to the fractions containing only ratios between odd numbers, conventional explanations have been forced to consider "composite fermions"[17] or in other words, a bunch of electrons. This is precisely the electrical displacement, or 'c'-rotation, of the various elements in the Reciprocal System.

Hence a number of associated phenomena, each one which requires a separate explanation in the standard theories due to their overlooking the presence of a 2D magnetic rotation, can be accounted for by this approach.

Ferromagnetism and Superconductivity

From the researches of Nehru, it is evident that both ferromagnetism[14] and superconductivity[18] are the results of combining 2D rotations, albeit in opposite directions. Ferromagnetism occurs when magnetism enters the time region, while superconductivity occurs when the two rotations get combined in the space region, which gives it the appearance of a double electron. The two occurrences are antagonistic to one another i.e. ferromagnetism requires the magnetic rotations to combine when they are aligned in the same direction, while in superconductivity the alignment has to be in opposing directions. Thus, in one case, the magnetism 'leaks out' as the magnetic field of a ferromagnet, while in the other case the magnetism 'implodes' to form a true magnetic current: the superconducting current. This explains why there is no resistance at all from the material in superconductivity, as the current is not electric at all. This 2D nature of the magnetic current is what has led to theories about "Cooper pairs of electrons."

It has been shown above that most of the ferromagnetic materials have a positive Hall Coefficient. If the relation between "holes" and 2D magnons is correct, then holes should have a strong correlation with superconductivity as well. It is interesting to see that it is definitely one of the top contenders[19]- which has led some researchers to associate superconductivity to the presence of holes or p-type conductivity (Fig. 5.)

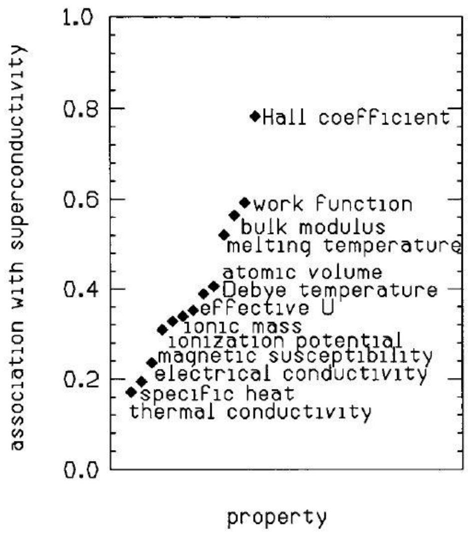

Fig. 5: Correlation of positive Hall coefficient with superconductivity

The overall relationship between the holes/magnons, ferromagnetism and superconductivity can be indicated as follows:

Ferromagnetism (2D in time region) 🡐 *'Holes' or 2D magnons* 🡒 *Superconductivity (2D in space region)*

Conclusions

The Hall Effect is still used today as a central tool in experimental material science, without realizing its importance for theory. Thus, there is sufficient experimental support to consider the magnetic rotation, its displacements and flow, to be as independent as electrons and electricity, although further research is naturally needed to clarify the details. They form a complementary system to the electrical components, and not a mutually antagonistic one, and clarify several puzzles that are inevitable when considering an electric-alone system. This bias of viewpoint, which has been highlighted with the help of the Reciprocal System, has to be removed for further research into solid state physics.

References:

[1] E. H. Hall, "On a New Action of the Magnet on Electric Currents," *Am. J. Math.*, Vol. 2, No. 3 (Sep., 1879), pp. 287-292
[2] G S Leadstone, "The discovery of the Hall effect," *Phys. Educ.* Vol 14 (1979), p. 378
[3] P. W. Bridgman, "Biographical Memoir of Edwin Herbert Hall," *National Academy of Sciences* Vol. XXI (1939), p. 84
[4] R. C. Tolman, T. D. Stewart, "Mass of electric carrier in copper, silver and aluminum," *Physical Review* Vol. 9 (1917) p. 164
[5] P. Drude, "Zur Elektronentheorie der metalle," *Annalen der Physik* 306 (3) (1900) p. 566
[6] R. E. Peierls, "Bird of Passage: Recollections of a Physicist," *Princeton University Press*, (1985) p. 37
[7] L. Hoddeson, E. Braun, et al, "Out of the Crystal Maze," *Oxford University Press* (1992) p. 112
[8] R. E. Peierls, "Zur Theorie des Hall-Effekts," *Phys. Z.* 30(1929) pp. 273-274
[9] D. B. Larson, "Basic Properties of Matter," Chapter 19 (1989) p. 210
[10] D. B. Larson, "Nothing But Motion," *North Pacific Publishers*, Chapter 9 (1979) p. 122
[11] D. B. Larson, "Basic Properties of Matter," Chapter 19 (1989) p. 216
[12] http://it.stlawu.edu/~koon/HallTable.html
[13] F. Seitz, "The Modern Theory of Solids," *McGraw Hill Publishers*, (1940) p. 183
[14] K.V.K. Nehru, "Is Ferromagnetism A Co-magnetic Phenomenon?," *Reciprocity*, XIX (1) (1990), p. 6
[15] N. Nagaosa, et al, "Anomalous Hall Effect," *Rev. Mod. Phys.* 82 (2010) p. 1539
[16] Liu, Chao-Xing et al, "The quantum anomalous Hall effect". arXiv:1508.07106 (2015-08-28)
[17] J. K. Jain, "Composite-fermion approach for the fractional quantum Hall effect," Phys. Rev. Lett. 63 (1989) p. 199
[18] K.V.K. Nehru, "Superconductivity: A Time Region Phenomenon," *Reciprocity*, XIX (3) (1990), p. 1
[19] J. E. Hirsch "Correlations between normal-state properties and superconductivity" Phys. Rev. B 55 (1997) p. 9007