A short note on the units of measurement

Abstract

This article summarizes the types of "dimensional maps" of units of measurement. It is shown that units of measurement form a type of basis set that is not unique. The proper understanding of the units might improve our understanding of physics as such.

1 Introduction

The classical and the modern physics relies on the possibility to measure things objectively. The types of measurement that were unrelated to one an other were given distinct units and these units represent a standard against which the measurement is being done. It was noted for the first time probably by Karl Friedrich Gauss that these units (in his system centimeter, gram, second) were orthogonal and created a complete system where all other units of measurement could be expressed as a product of these three. Later temperature (Celsius), luminous intensity (Candela), electric current (Ampere) and amount of substance (mole) would join them. Today most of the world uses the *System International* - SI (kg,m,s,K,Cd,mol) based on even older measurement system dating back probably to ancient Babylon derived from the earth size and it's rotation time.

It should be noted, that candela is not strictly a physical unit, because it reflects the behavior of the human eye and can be expressed only using a standard luminosity function as

$$I_v(\lambda) = 683.002\bar{y}(\lambda)I_e(\lambda) \tag{1}$$

Where $683.002\bar{y}(\lambda)$ is a standard luminosity function reflecting the preception of the human eye. When this factor is removed, it represents the power per steradian. "The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian."

Nonetheless Candela is not needed to completely describe the physical reality and therefore we will not consider it as a base unit in the following discussion.

2 Standard mapping

The SI system creates an exponential vector space where the units are supposed to be orthogonal and all other units should be derivable from it. The basis of this space is given by it's units - (kg,m,s,K,mol). Strictly speaking, this is not entirely true. The first indication can be found in the radian unit - which is supposed to be a derived unit. It is stated that a radian is $2\Pi m.m^{-1}$. Eventhough an angle is quite a specific measure, it is not considered to be a unit on it's own but is linearly dependent on a meter. This means it is not an "orthogonal" unit. The same we however find in the units that are supposed to be orthogonal. Namely Kelvin: The equation for the ideal gas

$$PV = nRT \tag{2}$$

where P is pressure V is volume, n is amount of substance, R is the universal gas constant and T is temperature, the temperature could be easily expressed in units of the gas constant and would become J/mol. So the temperature is a measure of energy per amount of the substance which is a derived unit. This is more evident in the statistical thermodynamics where according to equipartition principle each degree of freedom has an kinetic energy $k_B T/2$. Here the Boltzman's constant k_B is the rate of the universal gas constant and the Avogadro's constant

$$k_B = \frac{R}{A_r} \tag{3}$$

The Avogadro's constant expresses the number of entities in one mole and therefore can be considered as a linearly dependent unit to the "real" unit.

Thus we get a system where the basis is only given by only four units - kg,m,s,A.

3 Steinmetz mapping

Using the idea of the exponential vector space, we may change the basis and replace the existing unit set with another one. This is exactly what Charles Steinmetz did in his book "Electric discharges, Waves and Impulses". He replaced ampere and kilogram with linear combinations of the original four units: Weber (magnetic flux - $kg.m^2.s^{-2}.A^{-1}$) and Coulomb (electric charge - $A.s$). Same concepts mean the same thing but they have different dimensions - e.g. magnetic induction is the number of magnetic flux lines per square meter:

$$B = \frac{\Phi}{S}[Wb \times m^{-2}] \tag{4}$$

This made all electric relationships clear and understandable (shown in appendix 1). Further more this mapping directly shows that electromagnetism and mass are not separable from one another as kg would be expressed as

$$[kg] = [Wb \times C \times s \times m^{-2}] \tag{5}$$

In consequence the electric relations become clear while mechanical relations become just as obscure as the electric were in the standard mapping.

4 Larson mapping

We may ask whether we already identified all the linear dependencies there are. We can find a ratio between space and time using the speed of light

$$c = \frac{s}{t} \tag{6}$$

and a ratio between electric and magnetic field using Von Klitzing constant.

$$R_k = \frac{2\phi_0}{e} \tag{7}$$

This way we would get a mapping using only two quantities - one electromagnetic and one time-space quantity. This mapping was never practically used, but only demonstrates the possibility of such reduction.

The Larson mapping however is the one where space and time form the basis. The way to this mapping is quite indirect because as we saw, merging two units into one would result some other units than space or time.

Here we choose to demonstrate the Larson mapping based on similarity of behavior of mass and magnetism (this example was used by Eric Dollard in his lecture in 2014): If we move with a constant speed and then suddenly decide to increase our velocity, the force U we will experience will be the ratio between the change of our inertia $d\Phi$ per a change of time dt in that moment.

$$U = \frac{d\Phi}{dt} \tag{8}$$

Now if we change the the magnetic flux inside an electric circuit we get the voltage in that circuit, we can use the same relation even without change of the letters.

In Dewey B. Larson's Reciprocal System this relationship holds even without change of *units* [1]. From this we can see that for the relation between force and energy to hold, the electric current must have the dimensions $s.t^{-1}$ i.e. speed.

$$[kg \times m \times s^{-2}] = [kg \times m^2 \times s{-}2A^{-1}] \tag{9}$$

Consequently, the charge must be equivalent to the distance.

We can also find the similarity in inductance, that has the same relation with current to energy as mass has with the speed.

$$\frac{mv^2}{2} \equiv \frac{LI^2}{2} \tag{10}$$

and the magnetic flux is electromagnetic equivalent to the inertia.

$$p = m.v \equiv \Phi = L.I \tag{11}$$

Again in the reciprocal system these to quantities have the same dimensions.

For the next reduction we will need a similar analogy: a relation connecting time, space and mass has to be analogous to a relation connecting only time and space.

As electric current has no direction (not to be confused with current density, which does have direction), it is said to have "scalar speed" as opposed to "directional speed".

The other reduction of dimesions in the Larson's Reciprocal System is done via the equivalence of electric capacity to volume speed (dV/dt - often used in hydrodynamics).

$$C = \frac{\sigma}{dB/dt} \equiv \upsilon = \frac{P}{p} \tag{12}$$

[1] For completeness one should note, that Larson did not use deduction, to conclude the equivalence, he rather used inductive method to conclude it from the postulates of his theory. Further more, what we call "charge" he called "quantity of charge". He termed charge what we would call "energy stored in charge". Consequently he got other dimensions for electric concepts like inductance etc. so his usage of those concepts was different than we have and doesn't work with our picture. Quantities in appendix A also differ correspondingly

From this follows that the energy (e.g. $CU^2/2$) in Larson mapping has the dimensions $t.s^{-1}$ and the basis is only second and meter. This is extremely impractical, because expressing mass in $[s^3.m^{-3}]$ doesn't give any idea how much mass it should be. Larson himself used therefore cgs units or his own natural units.

5 Natural units

The other way to decrease number of basis vectors is to use some kind of natural units i.e. units that are bind to some universal "elementary" constant which is set to 1. Thus we can use the mole and connect it with charge by the Faraday constant and we get an elementary charge. This constant would represent a unit charge in any measuring system. Similarly we can couple this elementary charge with the elementary magnetic flux with the Von Klitzing constant:

$$\phi_0 = \frac{R_k e}{2} \tag{13}$$

and obtain a unit action:

$$h = 2\phi_0 e \tag{14}$$

Different types of natural units use different set of constants set to 1. For example atomic units use $m_e = \frac{1}{4\pi\epsilon} = \hbar = e = 1$ or Planck's units use $G = h = c = \frac{1}{4\pi\epsilon} = k_B = 1$. As we may see these four are linearly independent and generate basis and it means that units in the traditional sense disappear.

Larson used units where $c = R_\infty = e = m_{^{16}O}/16$. Combined together the reciprocal system uses only "displacemes" from the unit level.

It might be ineteresting to note, that in nature we observe charge, magnetic flux and action as truly quantized - i.e. there was no observation of a fractional part of these units to this day. On the other hand, there seems to be no (experimental) indication, that energy or frequency of are quantized.

6 Less units, more trouble?

In the process of decomposing the system of units we encounter a serious problem: each step down also leaves out some information about the "origin" of the quantity. The problem may be seen even on the "top level" in the SI units. They don't contain angle as a basic unit and therefore e.g. action takes the same units as moment of inertia while the two differ by 2Π - the converting factor. This situation is then similar to the conversation where we want to describe a distillation and crystallization of water without the words such as steam and ice. "We boil water to get water and cool it down to get again water. Then further cool it until we get water." (This originates from Phillip Porter's speach) Of course we can correct the sentence by describing steam and ice but the language will be too long to speak with and not feasible for the scientific use.

7 Conclusion

This article demonstrated how units of measurement are not a fixed thing but can be used to easily navigate through the concepts in the science. Therefore we may call the different systems of units (different basis sets) “dimensional maps”. When studying the basic relations in the universe such as light and matter, Larson’s units become a great tool as they show only the basic necessary relations and allow reasoning where more standard systems can’t, while SI is far best suited for everyday life or in the lab. Therefore it is not possible to expect wide usage of any kind of natural units in the common scientific practice. The situation is similar to the programming where the Assembly language is the most basic and theoretically can create any possible program, however in practice we use high level programing languages such as C++ for almost all applications.

Appendix A

In the following table one can see the relative complexity of the concepts used in one mapping or another. Clearly the Steinmetz mapping is better suited for electric engineering than any other and the standard mapping is best suited for mechanics. Larson’s mapping shows the equivalence between different concepts but also looses some information and thus becomes impractical for everyday use.

Quantity	Larson map	Steinmetz map	Standard SI map
Electric charge	m	C	$A.s$
Magnetic flux	$m^{-2}.s^2$	Wb	$kg.m^2.s^{-2}.A^{-1}$
Action	$m^{-1}.s^2$	$C.Wb$	$kg.m^2.s^{-1}$
Dielectric induction	m^{-1}	Wb/m^2	$m^{-2}.s.A$
Magnetic induction	$m^{-4}.s^2$	C/m^2	$kg.s^{-2}.A$
Current	$m^1.s^2$	$C.s^{-1}$	A
Voltage	$m^{-2}.s^1$	$Wb.s^{-1}$	$kg.m^2.s^{-3}.A{-}1$
Power	m^{-1}	$C.Wb.s^{-2}$	$kg.m^2.s^{-3}$
Work	$m^{-1}.s^1$	$C.Wb.s^{-2}$	$kg.m^2.s^{-2}$
Force	$m^{-2}.s^1$	$C.Wb.s^{-1}.m^{-1}$	$kg.m.s^{-2}$
Pressure	$m^{-4}.s^1$	$C.Wb.s^{-1}.m^{-3}$	$kg.m.s^{-2}$
Capacity	$m^3.s^{-1}$	$C.Wb^{-1}.s^1$	$kg^{-1}.m^{-2}.s^4.A$
Induction	$m^{-3}.s^3$	$C^{-1}.Wb.s^1$	$kg.m^2.s^{-2}.A^{-2}$
Impedance	$m^{-3}.s^2$	$C^{-1}.Wb$	$kg.m^2.s^{-3}.A^{-2}$